RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2017

THIRD YEAR [BATCH 2015-18]

MATHEMATICS [Honours]

Paper : VI

Date : 23/12/2017

Time : 11 am – 3 pm

Full Marks : 100

[Use a separate Answer Book for each Group]

Group - A

Answer any six questions from Question Nos. 1 to 9 : [6×5]

1. a) What is basic feature of Hermite interpolation? 2

   b) State Stirlings and Bessel's interpolation. When are they used? 2+1

2. Obtain numerical differentiation formula based on Lagrange's interpolation formula at an interpolating point. 5

3. Deduce Newton-Cote's formula for evaluation of $\int_a^b f(x)\,dx$. 5

4. a) Discuss the bisection method to find a simple root of an equation $f(x) = 0$. 3

   b) Prove or disprove: Bisection method is always convergent. 2

5. Establish Newton-Raphson method for a real root of an equation $f(x) = 0$. Give the geometrical interpretation of the method. 4+1

6. Stating clearly the pivoting process, write down the steps to be followed to solve the system of linear equations $AX = B$ where $A = (a_{ij})_{n\times n}$, $X = (x_1, x_2, \cdots, x_n)^t$ & $B = (b_1, b_2, \cdots, b_n)^t$ by Gauss elimination method. 5

7. Describe the power method to calculate numerically the greatest eigenvalue of real symmetric matrix of order n. 5

8. State Picard's recursion formula in connection with the solution of a first order differential equation $\frac{dy}{dx} = f(x, y),\ y_0 = f(x_0)$.

   Use Picard's method to compute $y(0.1)$ from the differential equation: $\frac{dy}{dx} = x + y,\ y = 1$ when $x = 0$. 2+3

9. Solve the equation $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$ by 4th order R-K. Method from $x = 0$ to $x = 0.2$ with step length $h = 0.1$. Compare the result with exact result (correct to 4 decimal places). 4+1

Answer any four questions from Question Nos. 10 to 15 : [4×5]

10. a) Use Green's theorem to evaluate the integral $\oint_C \left(x^2 dx + \left(x + y^2\right) dy\right)$ where C is the closed curve given by $y = 0$, $y = x$ and $y^2 = 2 - x$ in the first quadrant. 3

b) Show that $\iint_S \vec{r}\cdot\hat{n}\, dS = 3V$ by Gauss div. theorem, where V is the volume enclosed by the closed surface S. 2

11. a) Prove that $\bar{\nabla}\times\left(\vec{A}\times\vec{B}\right)=\left(\vec{B}\cdot\bar{\nabla}\right)\vec{A}-\vec{B}\left(\bar{\nabla}\cdot\vec{A}\right)-\left(\vec{A}\cdot\bar{\nabla}\right)\vec{B}+\vec{A}\left(\bar{\nabla}\cdot\vec{B}\right)$ 4

b) Show that any irrotational vector field is conservative. 1

12. Find the constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1,2,–1) has a maximum of magnitude 64 in a direction parallel to z axis. 5

13. a) If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A}\times\vec{B}$ is solenoidal. 2

b) Prove that $div\left\{r\ grad\ \left(r^{-3}\right)\right\} = 3r^{-4}$, where $\vec{r} = \left(x, y, z\right)$ and $r = \left|\vec{r}\right|$. 3

14. Evaluate $\iint_S \left(\bar{\nabla}\times\vec{A}\right)\cdot d\vec{S}$, where $\vec{A} = \left(x^2 + y - 4\right)\hat{i} + 3xy\hat{j} + \left(2xz + z^2\right)\hat{k}$ and S is the surface of the paraboloid $z = 4 - \left(x^2 + y^2\right)$ above the xy plane. 5

15. a) Evaluate $\oint\left(3x^2 + 2y\right)dx - \left(x + 3\cos y\right)dy$ around the parallelogram having vertices at (0,0), (2,0), (3,1) and (1,1). 3

b) A particle moves along the curve $x = t^3 + 1,\ y = t^2,\ z = 3t + 1$. Find the velocity and acceleration at t = 1, in the direction $\hat{i} + \hat{j} + 3\hat{k}$. 2

Group - B

Answer any two questions from Question Nos. 16 to 18 **:** [2×12]

16. a) Prove that the motion of a rigid body about its centre of inertia is the same as it would be had the centre of inertia were fixed and the same forces acted on the body. 7

b) A uniform elliptic board swings about a horizontal axis at right angles to the board and passing through one focus. If the centre of oscillation be at the other focus, then prove that the eccentricity of the ellipse is $\sqrt{2/5}$. 5

17. a) A uniform rod is held at an inclination α to the horizontal with one end in contact with the horizontal table whose coefficient of friction is μ. If it be then released, then show that it will commence to slide if $\mu < \dfrac{3\tan\alpha}{1+4\tan^2\alpha}$. 6

b) A rod of length $2a$ is suspended by a string of length l, attached to one end. If the string and the rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, then show that $\dfrac{3l}{a} = \dfrac{\left(4\tan\theta - 3\tan\phi\right)\sin\phi}{\left(\tan\phi - \tan\theta\right)\sin\theta}$. 6

18. a) A rigid body is rotating about a fixed axis. Find the expressions for

i) The K.E. of the rigid body

ii) The angular momentum about the axis of rotation. 7

b) A solid homogeneous cone, of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex; show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(4+\tan^2\alpha\right)$. 5

Answer <u>any one</u> question from <u>Question Nos. 19 & 20</u> : [1×6]

19. Three equal rods AB, BC, CD are freely jointed and placed in a straight line on a smooth table. The rod AB is struck at its end A by a blow which is perpendicular to its length. Show that the velocity of the centre of AB is 19 times that of CD and its angular velocity is 11 times that of CD. 6

20. A solid circular cylinder of radius a rotating about its axis is placed gently with its axis horizontal on a rough plane, whose inclination to the horizon is α. Initially, the friction acts up the plane and the coefficient of friction is μ. Show that the cylinder will move upwards, if $\mu > \tan\alpha$. Also show that the time that elapses before rolling commences is $\frac{a\Omega}{g\left(3\mu\cos\alpha-\sin\alpha\right)}$, where Ω is the initial angular velocity of the cylinder. 6

Answer <u>any two</u> questions from <u>Question Nos. 21 to 23</u> : [2×10]

21. a) A planet of mass M and periodic time T, when at its greatest distance from the sun comes into collision with a meteor of mass m, moving in the same orbit in the opposite direction with velocity v. If $\frac{m}{M}$ be small, show that the major axis of the planet's path is reduced by $\frac{4m}{M}\cdot\frac{vT}{\pi}\cdot\sqrt{\frac{1-e}{1+e}}$. 5

b) A smooth parabolic tube is placed, vertex downwards, in a vertical plane. A particle slides down the tube from rest under the influence of gravity. Prove that, in any position, the reaction of the tube is $\frac{2W(h+a)}{\rho}$, where W is the weight of the particle, ρ is the radius of curvature, $4a$ is the latus rectum and h is the original height of the particle above the vertex. 5

22. a) If ω be the angular velocity of a planet at the nearer end of the major axis, prove that its period is $\frac{2\pi}{\omega}\sqrt{\frac{(1+e)}{(1-e)^3}}$. 4

b) A spherical raindrop, of radius a, falls from a height h acquires moisture from the atmosphere throughout its motion, the radius thereby increases at the rate of ca. Show that when it reaches the ground its radius becomes $ca\sqrt{\frac{2h}{g}}\left(1+\sqrt{1+\frac{g}{2c^2h}}\right)$. 6

23. a) A rough cycloid has its plane vertical and the line joining its cusps is horizontal. A heavy particle slides down the curve from rest at a cusp and comes to rest again at the point on the other side of the vertex where the tangent is inclined at an angle of 45° to the vertical. Show that the coefficient of friction satisfies the equation $3\mu\pi+4\log\left(1+\mu\right)=2\log 2$. 6

b) A particle is projected from the Earth's surface with velocity v. Show that, if the diminution in gravity be taken into account, but the resistance of the air neglected, the path is an ellipse of major axis $\frac{2ga^2}{2ga-v^2}$, where a is the Earth's radius and v is such that the orbit is an ellipse. 4

——— × ———